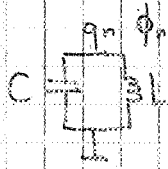


# 1st example: LC oscillator



$$i_c = C \frac{d}{dt} V_c \quad \frac{d\phi}{dt} = V_L = V_c$$

Kirchhoff:  $i_c = i_L = 0 \quad V_c = V_L$

$$\phi = \int v dt = L i_L = L I$$

$$q = \int i dt = C U$$

$$\mathcal{L}(q, \dot{q}) = \frac{q^2}{2C} - \frac{L}{2} \dot{q}^2$$

$$0 = \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{d}{dt} (L \dot{q}) + \frac{q}{C} - L \ddot{q} - \frac{1}{C} q$$

$$\mathcal{H}(q, \phi) = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} \frac{\phi^2}{L}$$

$$\hat{\mathcal{H}}(\hat{q}, \hat{\phi}) = \frac{1}{2} \frac{\hat{q}^2}{C} + \frac{1}{2} \frac{\hat{\phi}^2}{L}$$

harmonic oscillator

$$[\hat{q}, \hat{\phi}] = i\hbar$$

$$\hat{q} = \sqrt{\frac{\hbar}{2Z_c}} (a^\dagger + a)$$

$$\hat{\phi} = i \sqrt{\frac{\hbar Z_c}{2}} (a^\dagger - a)$$

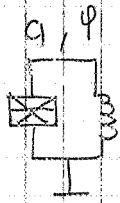
$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{q} + i \hat{\phi})$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{q} - i \hat{\phi})$$

$$\hat{\mathcal{H}} = \hbar \omega \left( a^\dagger a + \frac{1}{2} \right)$$

$$Z_c = \sqrt{\frac{L}{C}}$$

# RF-SQUID



$$\mathcal{H}(q, \phi) = \mathcal{H}_{\text{ind}} + \mathcal{H}_{\text{JJ}}$$

$$\mathcal{H}_{\text{ind}} = \frac{\phi^2}{2L}$$

$$\mathcal{H}_{\text{JJ}} = \frac{q^2}{2C} - E_J \cos(\delta)$$

$$V_J = \frac{\phi_0}{2\pi} \delta'$$

$$\phi = \int_{\partial D} V_{\text{ind}} d\tau$$

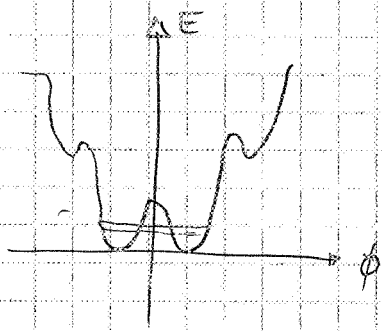
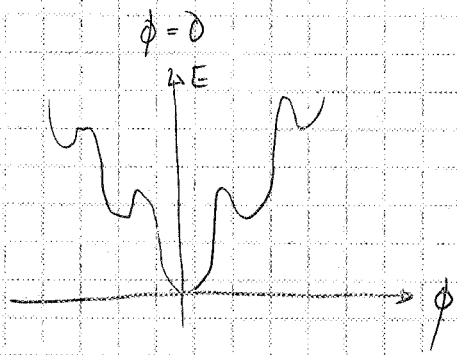
$$\phi_{\text{ext}} = \int \mathbf{v} = \phi - \underbrace{\int V_J d\tau}_{\frac{\phi_0}{2\pi} \delta}$$

$$\mathcal{H} = \frac{q^2}{2C} + \frac{\phi^2}{2L} - E_J \cdot \cos\left(\frac{2\pi}{\phi_0} (\phi - \phi_{\text{ext}})\right)$$

problem has 2 parameters

$$E_J/E_C \quad \frac{L_J}{L}$$

no analytical solution



problem: qubit very sensitive to flux noise

Slides

## Phase Qubit



current biased junction

$$\mathcal{H} = \frac{q^2}{2C} - I\phi - I\phi_0 \cos\left(\phi - \frac{2\pi}{\phi_0}\right)$$

$$S = 2\pi \frac{\phi}{\phi_0} \quad p = 2eq$$

$$\mathcal{H} = 4E_C p^2 - I\phi_0 S - I\phi_0 \cos(S)$$

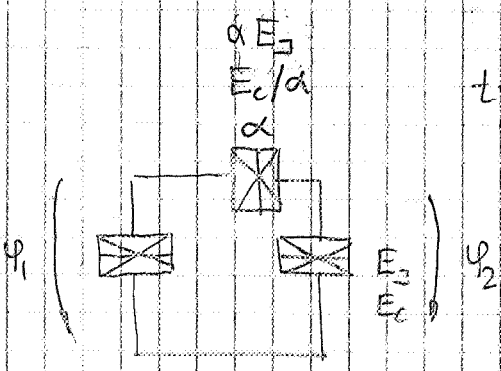
$$[S, p] = i$$

for  $I \approx I_0$

$$U(S) = \phi_0 (I_0 - I) (S - \pi/2) - \frac{I_0 \phi_0}{2\pi} (S - \pi/2)^3$$

$$\omega_p = \frac{1}{\sqrt{L_J(I)C}} = \frac{1}{\sqrt{L_J C_J}} \left(1 - (I/I_0)^2\right)^{1/4}$$

persistent-current quib

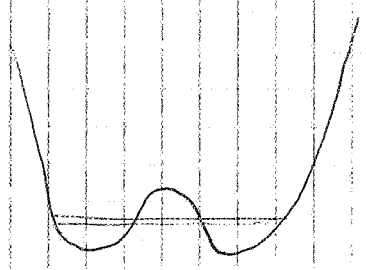


typical  $\alpha = 0,8$

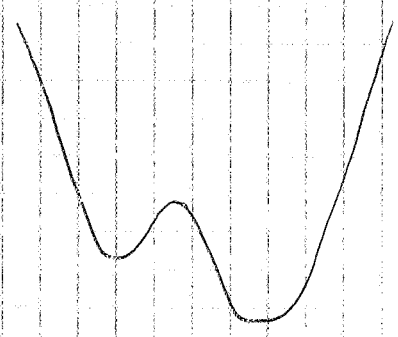
$$f = \frac{\phi}{\phi_0}$$

$$U = E_c \left[ -\cos \phi_1 - \cos \phi_2 + \alpha \cos (2\pi f + \phi_1 - \phi_2) \right]$$

$f = 0,5$

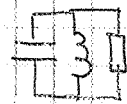
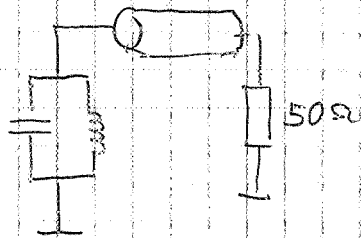
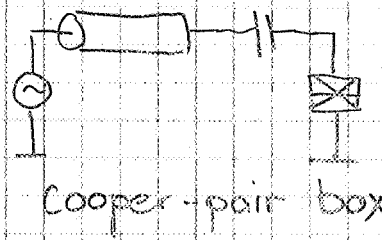
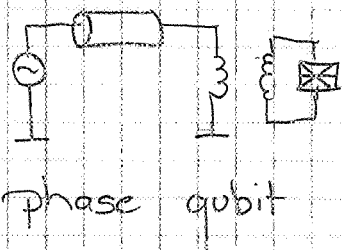
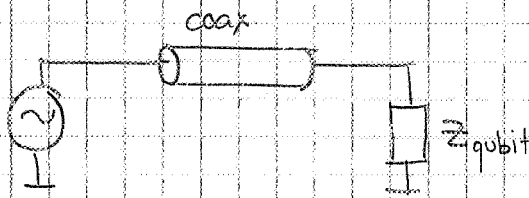


$f = 0,55$



# Qubit relaxation / dephasing

coupling to environment



RLC circuit

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\tau = \frac{1}{RC}$$

$$\propto e^{-\frac{t}{\tau}}$$

exponential decay

Fermi's golden rule

$$T = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 S$$

noise & dissipationrandom signal  $V(t)$ 

auto correlation

$$G_{VV}(t-t') = \langle V(t) V(t') \rangle = \langle V(t-t') V(0) \rangle$$

assumed stationary noise

$$\tilde{V}(\omega) = \int e^{i\omega t} V(t) dt$$

power spectral density

$$S_{VV}(\omega) = \langle |\tilde{V}(\omega)|^2 \rangle$$

since  $V(t)$  is real  
 $S_{VV}(-\omega) = S_{VV}(\omega)$

Wiener + Kinchin theorem

$$S_{VV} = \int_{-\infty}^{\infty} dt e^{i\omega t} G_{VV}(t)$$

example: white noise

$$G_{VV}(t) = \sigma^2 \delta(t)$$

$$S_{VV}(\omega) = \sigma^2$$

quantum noise

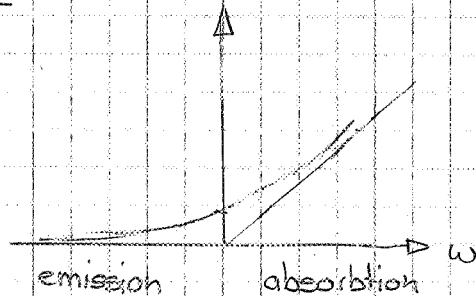
$$S_{XX}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \hat{x}(t) \hat{x}(0) \rangle$$

since  $\hat{x}$  is an operator

$$S_{XX}(-\omega) \neq S_{XX}(\omega)$$

quantum noise of a resistor

$$S_{vv}(\omega) = \frac{2R \hbar \omega}{1 - e^{-\hbar \omega / k_B T}}$$



for  $k_B T \gg \hbar \omega$  ('classical limit')

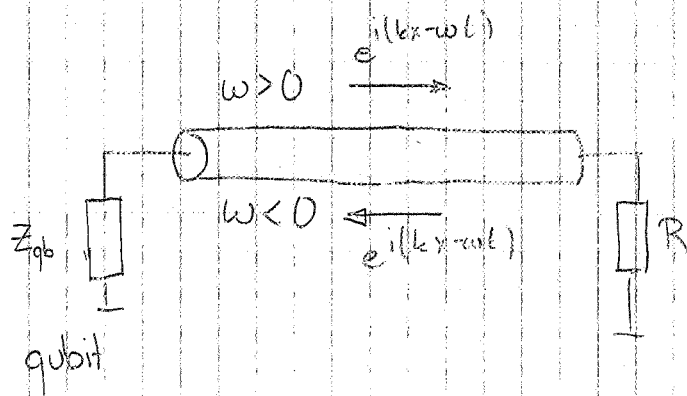
$$S_{vv}(\omega) = 2R k_B T$$

Johnson noise formula  $S_{vv}(\omega) + S_{vv}(-\omega)$

'quantum limit'

heavy-side function

$$S_{vv}(\omega) = 2R \hbar \omega \Theta(\omega)$$





## coupling to two-level system

$$H_0 = \frac{\hbar\omega_{01}}{2} \hat{\sigma}_z$$

$$H_1 = A F(\pm) \hat{\sigma}_x$$

perturbation theory, first order (Fermi's golden rule)

$$T_{\downarrow} = \frac{A^2}{\hbar^2} S_{FF}(\omega_{01})$$

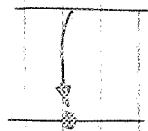
$$T_{\uparrow} = \frac{A^2}{\hbar^2} S_{FF}(-\omega_{01})$$

dephasing

$$T_{\phi} \propto S_{FF}(\omega \approx 0)$$

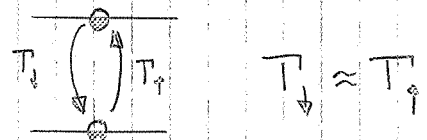
special form of the dissipation-fluctuation theorem

$$k_B T \ll \hbar\omega_0$$



qubit relaxes to ground state

$$k_B T \gg \hbar\omega_0$$



qubit is in an equal mixture of ground and excited state